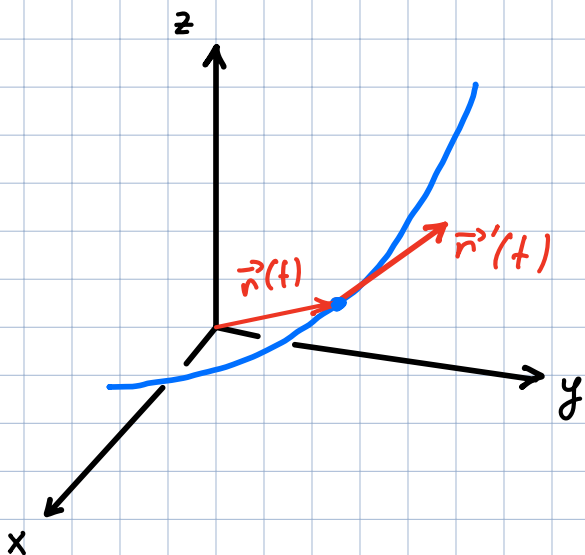


Last time: Motion in space



particle moving through space;
position at time t : $\vec{r}(t)$

$$\vec{r}'(t) = \lim_{h \rightarrow 0} \underbrace{\frac{\vec{r}(t+h) - \vec{r}(t)}{h}}_{\substack{\text{displacement vector} \\ \text{per unit time}}} = \vec{v}(t) \quad \uparrow \text{velocity vector}$$

speed: $|\vec{v}(t)| = |\vec{r}'(t)| = \frac{ds}{dt} \leftarrow \text{arc length fct.}$

$\uparrow$ rate of change of dist. w.r.t. time

Reminder: $s(t) = \int_a^t |r'(u)| du \Rightarrow \frac{ds}{dt} = |r'(t)|$

acceleration: $\vec{a}(t) = \vec{v}'(t) = \vec{r}''(t)$

Rmk: If we know $\vec{a}(t)$ and $\vec{r}(t_0), \vec{v}(t_0)$ (initial position and velocity) then

$$\vec{v}(t) = \int_{t_0}^t \vec{a}(u) du + \vec{v}(t_0), \quad \vec{r}(t) = \int_{t_0}^t \vec{v}(u) du + \vec{r}(t_0)$$

TRUE OR FALSE?

1. $\vec{T}(t)$ and $\vec{r}'(t)$ are parallel

2. $\vec{T}(t) = \vec{N}(t) \times \vec{B}(t)$

3. $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$

4. $\vec{N}(t) = \vec{B}(t) \times \vec{T}(t)$

5. $\vec{B}(t) = \frac{\vec{r}'(t) \times \vec{r}''(t)}{|\vec{r}'(t) \times \vec{r}''(t)|}$

6. $\vec{r}''(t) \cdot \vec{B}(t) = 0$

TRUE OR FALSE?

1. $\vec{T}(t)$ and $\vec{r}'(t)$ are parallel

TRUE

2. $\vec{T}(t) = \vec{N}(t) \times \vec{B}(t)$

TRUE

3. $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$

TRUE

4. $\vec{N}(t) = \vec{B}(t) \times \vec{T}(t)$

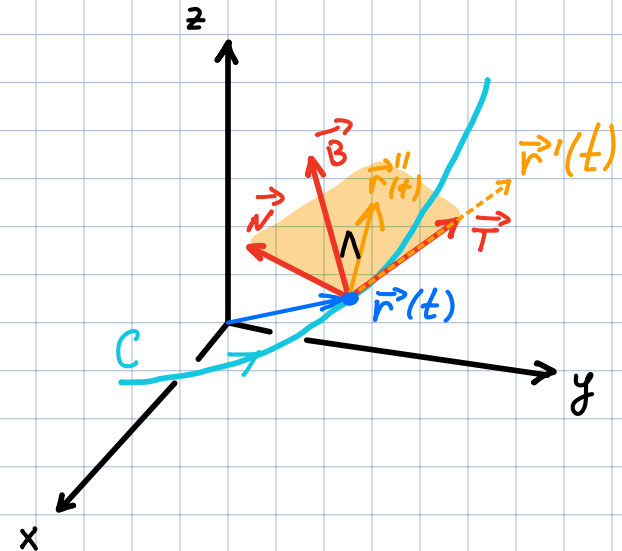
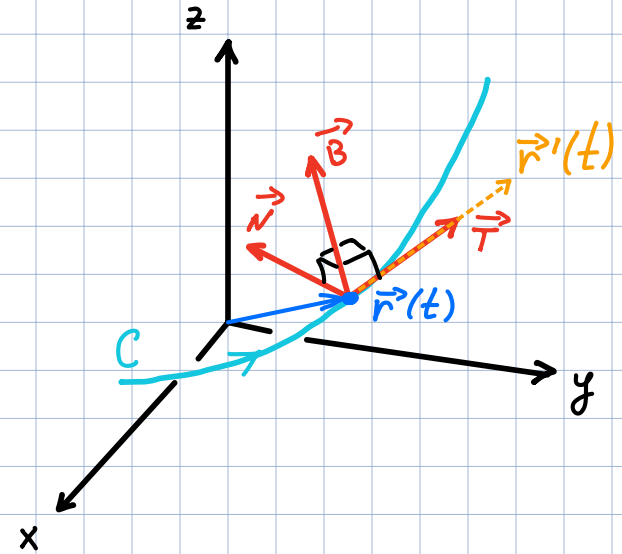
TRUE

5. $\vec{B}(t) = \frac{\vec{r}'(t) \times \vec{r}''(t)}{|\vec{r}'(t) \times \vec{r}''(t)|}$

TRUE

6. $\vec{r}''(t) \cdot \vec{B}(t) = 0$

TRUE



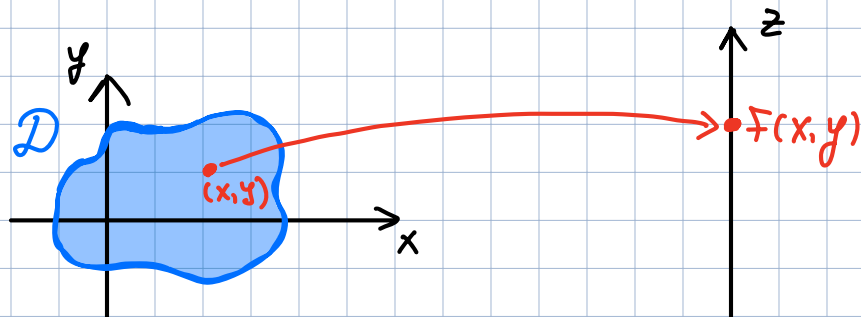
Today: Functions of several variables

Function of two variables:

$$f: (x, y) \mapsto f(x, y)$$

in $D \subset \mathbb{R}^2$ in $\mathbb{R}$

↑ domain



range: set of all values of $f(x, y) \in \mathbb{R}$

domain: all (x, y) s.t. $f(x, y)$ is well-defined.

Ex: $F(x,y) = \frac{\ln(y-x^2)}{y-1}$

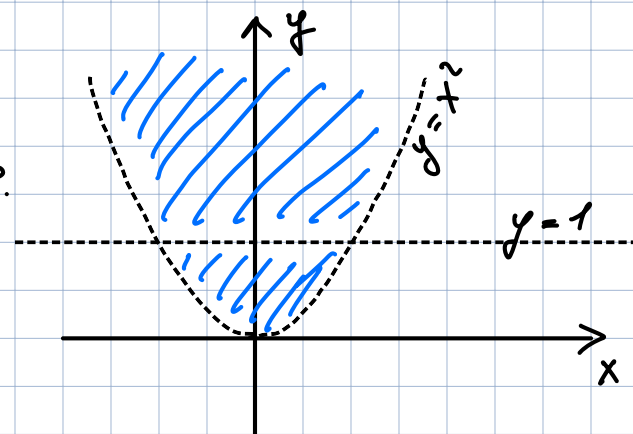
a) Find $F(1,3)$

b) Find the domain $\mathcal{D}$

Sol: (a) $F(1,3) = \frac{\ln(3-1^2)}{3-1} = \frac{\ln 2}{2}$

(b) domain = all (x,y) s.t. $f(x,y)$ is well-defined, i.e.

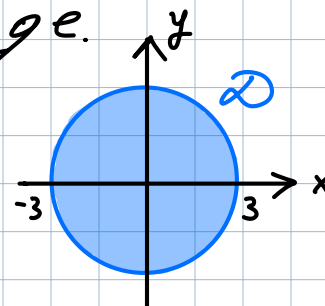
$$\mathcal{D} = \left\{ (x,y) : \begin{cases} y-x^2 > 0 \\ y-1 \neq 0 \end{cases} \right\}$$



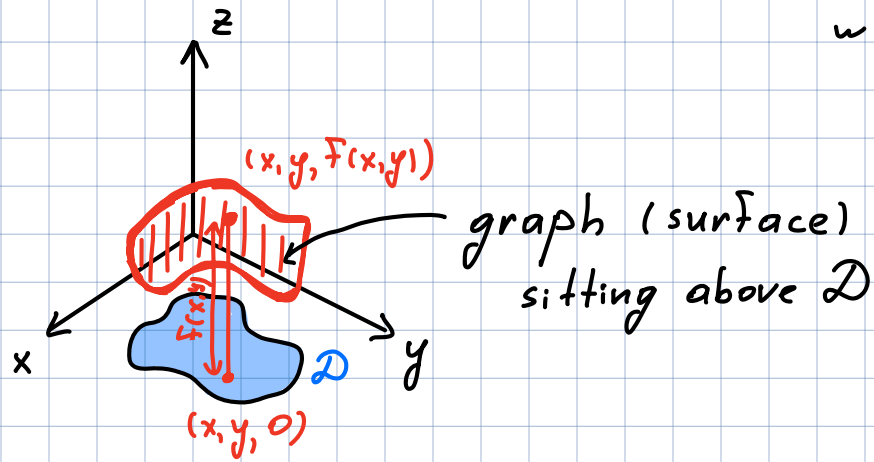
Ex: $F(x,y) = \sqrt{9-x^2-y^2}$. Find domain and range.

Sol: Domain: $9-x^2-y^2 \geq 0 \Leftrightarrow x^2+y^2 \leq 9$

Range: $\{z \mid z = \sqrt{9-x^2-y^2}, (x,y) \in \mathcal{D}\} = [0,3]$



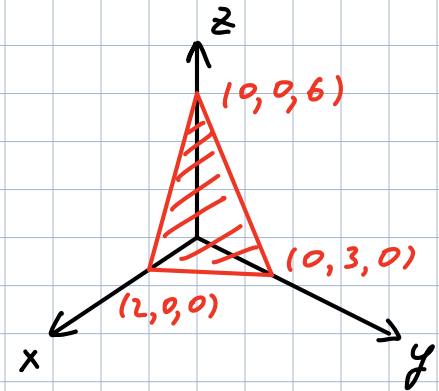
Graph: Set of points (x, y, z) in $\mathbb{R}^3$
with $(x, y) \in \mathcal{D}$, $z = f(x, y)$



Ex: $f(x, y) = 6 - 3x - 2y$; sketch the graph.

Sol: $z = 6 - 3x - 2y$ or $3x + 2y + z - 6 = 0$ - plane

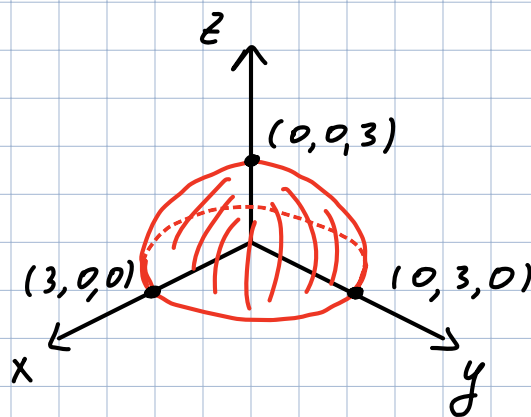
Intercepts: $y = z = 0 \Rightarrow x = 2$
 $x = z = 0 \Rightarrow y = 3$
 $x = y = 0 \Rightarrow z = 6$



Rmk: Generally: $f(x, y) = ax + by + c$ - linear function,
graph is a plane. ($z = ax + by + c$)

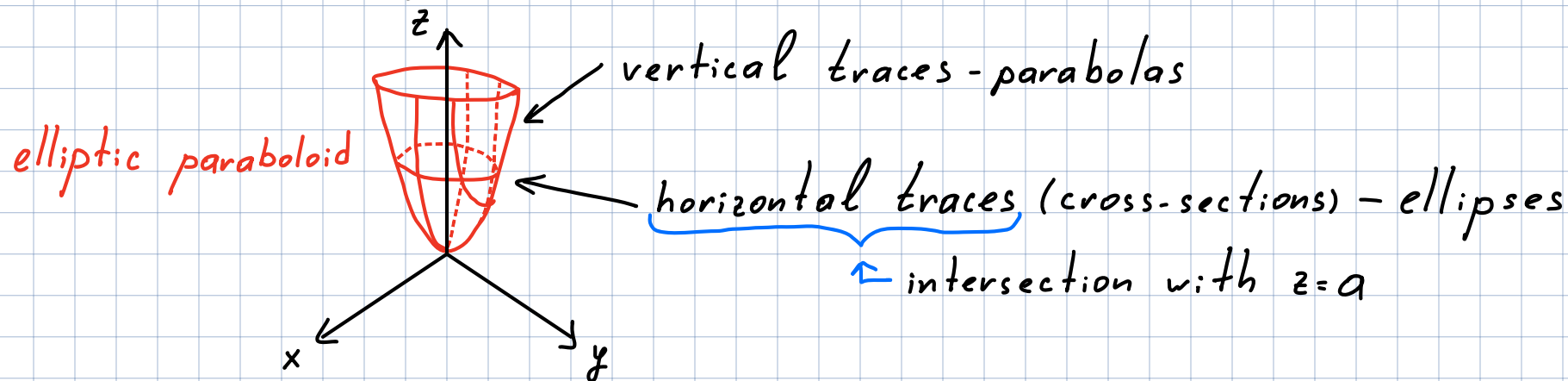
Ex: Graph $F(x,y) = \sqrt{9-x^2-y^2}$

Sol: $z = \sqrt{9-x^2-y^2} \Rightarrow z^2 = 9-x^2-y^2 \Rightarrow x^2+y^2+z^2 = 9$
 $\Downarrow$
 $z \geq 0$ and $z \geq 0$ } upper semi-sphere



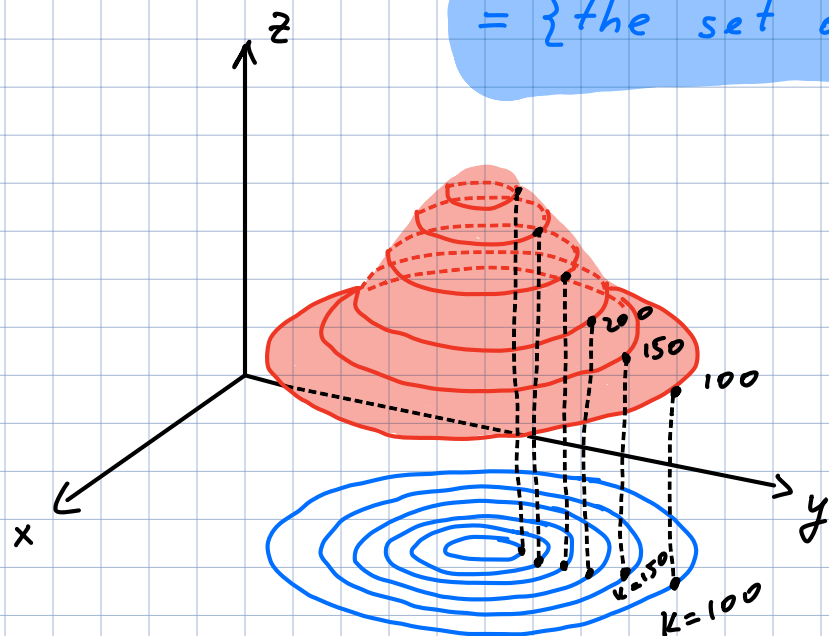
Ex: $h(x,y) = 4x^2 + y^2$. Find domain, range, graph.

Sol: $D = \mathbb{R}^2$, range = $[0, \infty)$



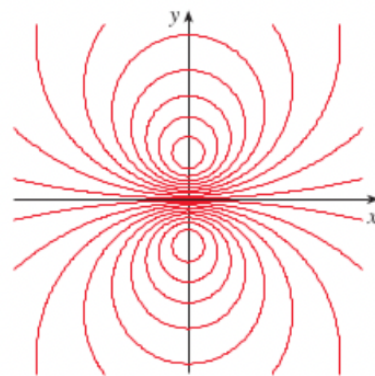
Level curves: curves with eq. $f(x,y) = k$ constant in range
 (or contour)

$= \{ \text{the set of all points } (x,y) \text{ s.t. } f(x,y) = k \}$

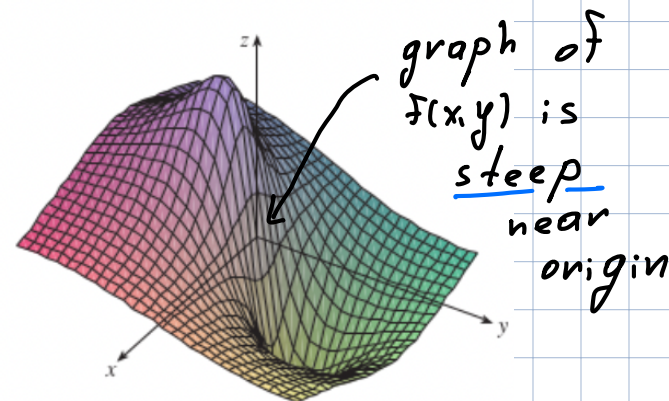


on this curve $f(x,y)=100$

level curves
are dense
near origin

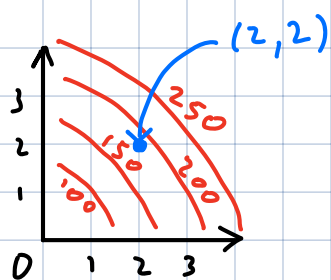


Level curves of $f(x,y) = \frac{-3y}{x^2 + y^2 + 1}$



$f(x,y) = \frac{-3y}{x^2 + y^2 + 1}$

Ex:



Estimate $F(2, 2)$

Sol: $F(2, 2) \approx 175$

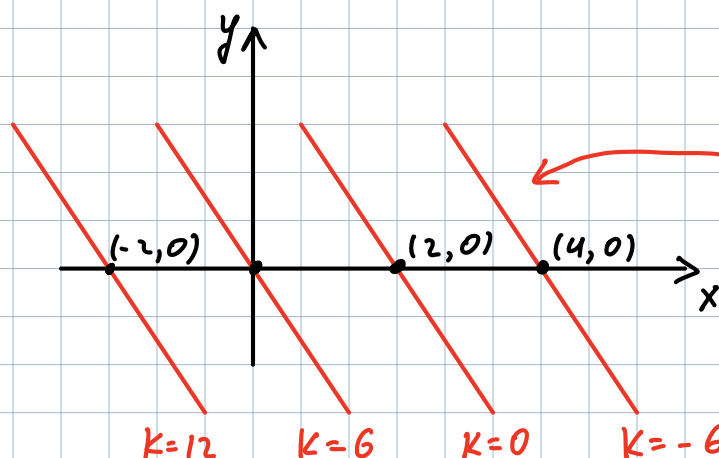
Ex: $F(x, y) = 6 - 3x - 2y$; sketch level curves for $k = -6, 0, 6, 12$

Sol: $6 - 3x - 2y = k$ or $3x + 2y + (k - 6) = 0$

↑ lines with slope $-\frac{3}{2}$

// $3x + 2y + (k - 6) = 0 \Leftrightarrow$

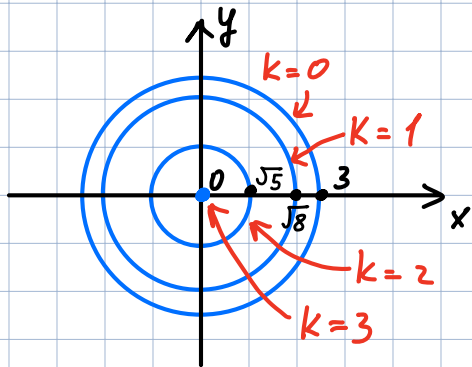
$$y = -\frac{3}{2}x + \frac{6 - k}{2} //$$



← equally spaced parallel lines

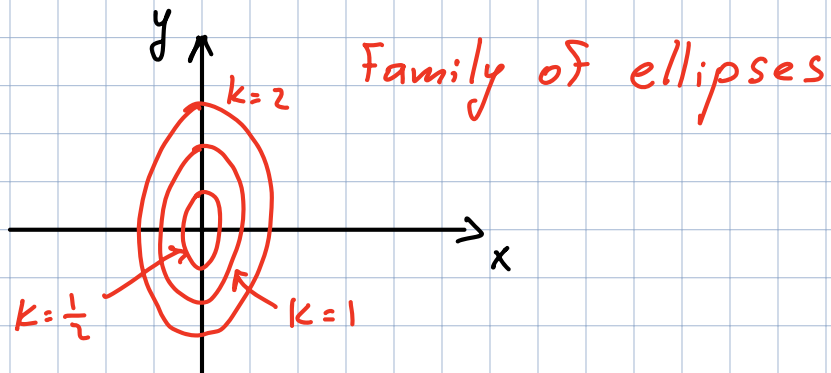
Ex: $f(x,y) = \sqrt{9-x^2-y^2}$, sketch level curves for $k=0,1,2,3$

Sol: $\sqrt{9-x^2-y^2} = k \Rightarrow 9-x^2-y^2 = k^2 \Rightarrow x^2+y^2 = 9-k^2$
 $\uparrow$ circle of radius $\sqrt{9-k^2}$



Ex: $h(x,y) = 4x^2 + y^2$, sketch level curves

Sol: level curves: $4x^2 + y^2 = k \Rightarrow \frac{x^2}{\frac{1}{4}k} + \frac{y^2}{k} = 1$

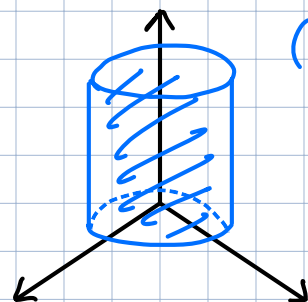


Functions of 3 variables:

$$f(x, y, z)$$

Ex: $f(x, y, z) = \sqrt{1-x^2-y^2} + \ln z$, find domain $\mathcal{D}$

Sol: $\mathcal{D}: \begin{cases} x^2 + y^2 \leq 1 \\ z \geq 0 \end{cases}$



(interior of)
upper half of
the cylinder

The early-semester feedback surveys
(should be distributed to you
this Friday)

PLEASE PARTICIPATE!